RESEARCH ON THE WARPING DEFORMATION IN FUSED DEPOSITION MODELING

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ABSTRACT
This paper focused on the parts warping deformation in FDM, analyzed the source of the deformation and its action mechanism. A mathematical model of warping deformation in FDM is established base on three hypotheses. Through the analysis of the prototyping process of FDM, the influence degree of prototyping parameters, including the number of layers, the section length, the shaping room temperature and the line shrinking coefficient are given comprehensively and quantitatively. The analysis results explained some phenomenon in FDM reasonably. Furthermore, the results of this paper are useful in decreasing the warping deformation and improving the quality of FDM products.

KEYWORDS
Rapid prototyping, Additive manufacturing, Fused deposition modeling and Warping deformation.

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INTRODUCTION
Among the most widely used and rapidly growing rapid prototyping or additive manufacturing (AM) technologies are extrusion deposition processes such as fused deposition modeling FDM®, fused filament fabrication and melt extrusion manufacturing MEM. FDM is one of the most common techniques on rapid prototyping and manufacturing RPM that develops in recent years. With the rapid development of modern industry, FDM is widely used in new concept product development, rapid mold manufacturing, making prototype of human organs, design and development of medical devices and so on. FDM becomes an important direction of development. However, the development is limited by the lower accuracy and intensity of FDM.
In FDM, the internal stress caused by volume shrinking will influence the prototyping dimension accuracy. Also, this may cause the warping deformation of prototypes. This phenomenon also exists in many other rapid prototyping technologies. In LOM (Laminated Object Manufacturing), the process parameters such as roller temperature, velocity and indentation are related to temperature and stress distribution with the laminate, which may lead to deformation\(^2\). Shrinkage and beam offset are the two most important control parameters in SLS (Selective Laser Sintering) process\(^3\). In\(^4\), some new concepts such as shrinkage force and surface-distributed loads are proposed according to the resin shrinkage while curing and the adhesive characteristics between adjacent layers in SL (Stereo lithography).

The warping deformation is the main reason for poor prototyping quality. To improve the quality of prototyping is one of the most urgent issues to be addressed in rapid prototyping\(^5\). However, FDM technology has its own particularity. To decrease the effects of internal stress on prototyping quality, a mathematical model of warping deformation must be established, which can provide the theoretical references for FDM technology as well as other RP technologies.

This paper focuses on the warping deformation in FDM process, and analyzes the influence degree of the printing parameters, so as to improve the prototyping quality. The rest of the paper continues as follows, Section 2 modeled the warping deformation in FDM process theoretically based on four hypotheses. Section 3 analyzed the warping deformation from four aspects, which is the number of layers, the section length, the shaping room temperature and the line shrinking coefficient, experimentally. In section 4, two suggestions for reducing warping deformation of FDM is given. In section 5, 108 mm×40 mm×29mm wing center link part of UAV is printed by Flash forge FDM printer as an example. Section 6 is the conclusion.

**MATERIAL AND METHODS**

**Theoretical model**

**Hypotheses**

As for the accumulation process of the amorphous thermoplastic plastics, some hypotheses must be made before the mathematical analysis of the internal stress and deformation.

The high temperature wires extruded from the nozzles deposits on the platform according to the predetermined trajectory, at the same time, the temperature of half molten wires drops from fusion temperature \(T_m\) to glass transition temperature \(T_g\). In this process, a certain volume shrinkage occurs on the wires, and in this temperature range, a large deformation of thermoplastic plastics may occurs under a small force, namely, its ability of resistance to external forces is weak\(^1\). Therefore, although volume shrinkage occurs, there is little internal stress gathered in wires. The internal stress in wires generates mainly in the period of the glass transition temperature \(T_g\) and the shaping room temperature \(T_e\).

The extruded high temperature elongated wires could be cooled below glass transition temperature \(T_g\) in a short time, taking ABS P400 as an example, supposing that the wires are extruded from a nozzle, whose aperture is 0.254 mm, it took about 0.55s for wires to cool down from 270°C to 94°C (glass transition temperature), 1.2s for wires to cool down from 94°C to shaping room temperature 70°C\(^6\). If the deposition velocity is 30 mm/s, the times that deposit a single layer of a medium-scale part is far longer than 1.75s. For this reason, we can consider that the temperature of the platform, deposited prototype and the shaping room are the same. And the wires just extruded have no heat loss, which means they have the same temperature as nozzles.

The prototype deformation due to internal stress releasing is the composition of deposited wires deformation of layer-to-layer and in-layer. When establishing the layer-to-layer deformation model, it is simplified that every layer is considered as a plate model, and the wires in a layer is deposited instantaneously. When the in-layer deformation is
analyzed, supposing that wires are routed as a 'Z' path that there is no cavity between wires.

**Warping model**

**Layer-to-layer deformation model**

In hypothesis (1), there is no internal stress in new extruded wires in the process of extruded temperature $T_0$ to glass transition temperature $T_g$, however, it is mainly generated in the process of $T_g$ to shaping room temperature $T_e$. The stress-deformation analysis process is in Figure No.1. Figure No.1a, is the perfect state with no warping deformation. But in reality, warping deformation exists. Supposing that the layer shrinks freely during the cooling process. At the same time, it is separated from the deposited plane, which is shown in Figure No.1b, so the line shrinkage of per unit length in layers is

$$\varepsilon = \frac{z - d}{E}$$

**From hypotheses (1) and (2), we can know that $\Delta T$ is a step function**

$$\Delta T = \begin{cases} T_g - T_e & s \leq z \leq h \\ 0 & 0 \leq z \leq s \end{cases}$$

The sum of internal stress after prototyping is 0, and the resultant moment is 0

$$\int_0^h [ -E\alpha \Delta T + \sigma + \frac{E(z-d)}{\rho} ] dz = 0$$

$$\int_0^h [ -E\alpha \Delta T + \sigma + \frac{E(z-d)}{\rho} ] zdz = 0$$

These two independent equations contains $d$, $\rho$ and $\sigma'$ unknown variables, so

$$\sigma' = \sigma - \frac{Ed}{\rho}$$

And Eq. (4) and (5) can be simplified as

$$\int_0^h [ -E\alpha \Delta T + \sigma' + \frac{Ez}{\rho} ] dz = 0$$

$$\int_0^h [ -E\alpha \Delta T + \sigma' + \frac{Ez}{\rho} ] zdz = 0$$

According to Eq. (7) and (8), the warping radius is

$$\rho = \frac{l^3}{6\alpha(T_g - T_e)(1-s)\rho}$$

As for FDM, the difference between $h$ and $s$ is the thickness of layer $\Delta s$, and for a n-layer prototype, $s/l = (n-1)/n$, so

$$\rho = \frac{l^3}{6\alpha(T_g - T_e)(1-s)s} = \frac{l}{6\alpha(T_g - T_e)(1-s)s}$$

$$= \frac{l}{6\alpha(T_g - T_e)(n-1)/n} = \frac{n\Delta s}{6\alpha(T_g - T_e)(n-1)/n^2}$$

According to Figure No.2,

$$\cos \theta = \frac{\rho - \delta}{\rho}$$

$$\rho\theta = \frac{L}{2}$$

(11)

So $\delta$ can be expressed as

$$\delta = \frac{n^3\Delta s}{6\alpha(T_g - T_e)(n-1)} \times \left\{ 1 - \cos \left[ \frac{3\alpha L}{n\Delta s (T_g - T_e)(n-1)/n^2} \right] \right\}$$

Where

$n$ - The number of layers

ρ - The warping radius
L - The section length of prototype
δ - The largest warping deformation
Δs - The thickness of layer

**In-layer deformation model**

According to hypothesis (3), the prototype deformation due to heat internal stress releasing is the composition of deposited wires deformation of layer-to-layer and in-layer. Therefore, the whole prototype deformation model can be obtained by the combination of the strain of layer-to-layer and in-layer.

When the in-layer deformation is analyzed, supposing that wires are routed as a 'Z' path that there is no cavity between wires. The analysis process is similar to layer-to-layer analysis, so the equations in 1.2.1 are also applicable.

However, in reality, the anisotropy of prototype can be decreased by choosing reasonable depositing paths in FDM, such as the stagger depositing paths of different layers and different beginning direction of layers. Eventually, the in-layer deformation can be ignored in the whole prototype deformation model.

**The analysis of warping deformation model**

From the layer-to-layer warping deformation model, it can be seen that the main impact coefficient that effects the warping deformation includes the number of layers n, the section length of prototype L, line shrinking coefficient α, the shaping room temperature $T_e$ and the glass transition temperature $T_g$, these coupled coefficients influences the heat deformation of prototype together.

To analyze the warping deformation model, ABS P400 wires, which is developed by Stratasys Company, is used. It is made of 90%-99% ABS resin, 0%-2% mineral oil and 0%-2% paraffin’s. The mainstream FDM printers are using ABS P400 as the raw material. The softening melting range of ABS P400 is wide (105-290°C), and its line shrinking rate is small (0.3%-0.5%)^7,8, the heat stability and chemical stability are fine, its strength is also big.

Generally, $T_g$ of ABS P400 is 94°C, and its density is 1150 kg/m$^3$, and the physical properties of ABS is given in Table No.1^9, the $T_s$ of the Flash forge FDM printer is 210°C, $T_e$ is 75°C, and its radius of nozzle is 0.2 mm.

Flash forge® FDM printer is used in this experimental analysis (see in Figure No.4). According to the above characteristic parameters and the Flash forge® FDM printer's working parameters, the heat deformation model is analyzed, and the influence degree of these parameters is given in the following 4 aspects.

**The influence of the number of layers n**

In Figure No.5, these three curves shows the deformation of 20 mm-section length, 70 mm-section length and 120 mm-section length, respectively. It can be seen that the warping of prototype decreases with n increasing, and when n is 18, the deformation flattens out, which means more number of layers has less effect on deformation. At the same time, if the section length is long, the deformation will be big. This regulation is helpful to choose a reasonable ratio of the section length and depositing height.

**The influence of the section length L**

In Figure No.6, these three curves represents the deformation of 20 layers, 70 layers and 120 layers, respectively. From Figure No.6 we can see that as the section length increases, the deformation also increases, and this trend becomes obvious as section length increasing. In the meantime, the deformation decreases with n increasing. Therefore, in FDM, we must shorten the length of wires in depositing direction as possible as we can, which means it is important to choose reasonable depositing directions and division strategies.

**The relationship between section length L and the deposition thickness n/n Δs under the same deformation**

Making $\alpha$ is 0.4%, $T_e$ is 75°C. From the prototyping parts, the relationship between section length and the deposition thickness under different warping deformation is given in Table No.2. From Table No.2 we can see that when the warping deformation is fixed, the ratio of section length and deposition thickness is approximately linear, which means the section length is proportional to the deposition thickness of prototyping parts with the same warping deformation. Hence, we can design a reasonable ratio of section length.
length and deposition thickness for parts according to Table No.2 in order to reduce the warping deformation of prototyping parts.

**The influence of the shaping room temperature** $T_e$

Making $\alpha$ is 0.4 %, $L$ is 70 mm, and $n = 20,70,120$, respectively, $T_e$ goes from 50°C to 94°C. It is shown in Figure No.7 that the deformation decreases approximately linearly. When the shaping room temperature arrives at $T_g$, the deformation is nearly 0. However, with the shaping room temperature increasing, the curing time becomes longer, and the depositing wires will adhere with the deposited wires, which may cause the failure of prototyping. So, there are different best $T_e$ for different materials. The best $T_e$ of ABS P400 is around 75°C.

**The influence of the line shrinking coefficient** $\alpha$

Making $L$ is 70 mm, $T_e$ is 75°C, and $n = 20,70,120$, respectively, $\alpha$ goes from 0.3% to 1%. It can be seen from Figure No.8 that the deformation increases approximately linearly. So, in FDM, it is useful for decreasing warping deformation and internal stress of prototype to adopt low line shrinking rate materials.

**The strategies for decreasing warping deformation of prototype**

From the above analysis, this paper gives two suggestions for reducing warping deformation of FDM.

**Material**

From Eq. (12) we can know that $T_e$ and $\alpha$ of materials influence the warping deformation directly. Decreasing $T_g$ and $\alpha$ of materials appropriately can reduce the deformation of prototype to some degree.

As for most thermoplastic plastics, their elasticity modulus is sensitive to temperature. Hence, raising the shaping room temperature to lower the internal stress could be helpful to reduce the warping deformation. It can be seen in Figure No.7.

**Prototyping techniques**

From Figure No.5 and Figure No.6, it can be seen that the dimension of prototypes is an important influencing factor, the larger the dimension is, especially the depositing length, the larger the warping deformation is. Hence, to avoid long thin prototypes as far as possible is a way to reducing warping deformation.

From Figure No.9, it can be seen that the deformation increases with the section length increasing, which can be proved in actual prototyping process. In Figure No.9, in the process of depositing two same thin board parts, the deformation of the part whose depositing wires is along its long side is far bigger than the other one. Therefore, to reduce the warping deformation, we can divide the parts into several slender pieces, and let the wires deposit along their short side.

In the process of prototyping, we find that the actual deformation is a little smaller than theoretical value. One reason is that before parts are deposited, substrates are deposited on the platform of Flash forge FDM printer, these substrates are coherent to the platform firmly, so they do not have deformation. The other reason is the support structures in FDM restrain the parts.

**Application example**

Using the analyzing results, a 108mm×40mm×29mm wing-center-link part of UAV is manufactured by Flash forge FDM printer, in Figure No.10. Adopt ABS P400 whose line shrinking rate is 0.4% as the depositing material, set the shaping room temperature as 65°C, and choose a reasonable beginning direction of layers according to the depositing length and the number of layers. Finally, the deposited part is shown in Figure No.11. Then measure the dimension of the wing-center-link part by API Tracker III laser tracker. The maximum deformation is 0.927mm, which meets the technical requirement.
Table No.1: The physical properties of ABS

<table>
<thead>
<tr>
<th>S.No</th>
<th>Temperature (°C)</th>
<th>Elasticity modulus (Pa)</th>
<th>Poisson’s ratio</th>
<th>Density (kg/m²)</th>
<th>Coefficient of thermal expansion</th>
<th>Coefficient of heat conduction</th>
<th>Specific heat (J/kg.K)</th>
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<tr>
<td>1</td>
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<td>3.9E9</td>
<td>0.38</td>
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<td>3E-2</td>
<td>1470</td>
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<tr>
<td>2</td>
<td>50</td>
<td>3.5E9</td>
<td>0.38</td>
<td>1150</td>
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<td>3E-2</td>
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<td>3</td>
<td>100</td>
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<td>1150</td>
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<tr>
<td>4</td>
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<td>1.68E9</td>
<td>0.38</td>
<td>1150</td>
<td>8.5E-5</td>
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<tr>
<td>5</td>
<td>200</td>
<td>1.0E9</td>
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<td>1150</td>
<td>8.5E-5</td>
<td>3E-2</td>
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<tr>
<td>7</td>
<td>300</td>
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<td>0.38</td>
<td>1150</td>
<td>8.5E-5</td>
<td>3E-2</td>
<td>1470</td>
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<tr>
<td>8</td>
<td>350</td>
<td>0.001E9</td>
<td>0.38</td>
<td>1150</td>
<td>8.5E-5</td>
<td>3E-2</td>
<td>1470</td>
</tr>
</tbody>
</table>

Table No.2: The relationship between section length and deposition thickness when deformation is fixed

<table>
<thead>
<tr>
<th>S.No</th>
<th>The warping deformation n δ:mm</th>
<th>The ratio of section length and the number of layers (L/n)</th>
<th>The ratio of section length and the deposition thickness (L/nΔs)</th>
</tr>
</thead>
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<tr>
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<td>0.2</td>
<td>0.7145768 0.7059863 0.7214785</td>
<td>2.8369478 2.8574156 2.865471</td>
</tr>
<tr>
<td>3</td>
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<td>4.7515102 4.7654891 4.7356987</td>
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<tr>
<td>5</td>
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<td>6</td>
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<td>1.6045744 1.6248972 1.6359478</td>
<td>6.4258952 6.4418798 6.4593201</td>
</tr>
</tbody>
</table>

(a)

(b)
Figure No.1: The prototype deformation due to wires volume shrinkage

Figure No.2: The relationship between the largest warping deformation and the warping radius

Figure No.3: The stagger depositing paths of different layers and different beginning direction of layers
Figure No.4: Flash forge® FDM printer

Figure No.5: The influence of number of layers $n$ on the warping deformation

Figure No.6: The influence of section length $L$ on the warping deformation

Figure No.7: The influence of shaping room temperature $T_e$ on the warping deformation
Figure No.8: The influence of line shrinking coefficient \(\alpha\) on the warping deformation

Figure No.9: The depositing paths along long side and short side

Figure No.10: The stl model of a 108mm×40mm×29mm wing-center-link part of UAV

Figure No.11: The manufactured wing-center-link part by Flash forge FDM printer
CONCLUSION
The warping deformation is one of the most important factors that influence the FDM quality. The reason for deformation is complex, including not only the characteristic of materials and the coefficients of FDM printers, but also the geometric structure of parts and depositing direction. In this paper, the theoretical analysis model is established by reasonable hypotheses, and the influencing degree of different factors, including number of layers, section length, shaping room temperature and the line shrinking rate of materials, are given quantitatively. And the strategy for reducing the warping deformation is introduced in materials aspect and prototyping technology aspect. It also provides the references for reducing the warping deformation in FDM. In the end, a part of UAV is printed, and the result is satisfied.

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CONFLICT OF INTEREST
Authors declare no conflict of interest.

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